Therefore

$$0=u_0=u_2=u_4=\ldots$$

That is, *if a locus symmetrical with respect to the origin contains one term of odd degree, it contains no absolute term and no terms of even degree.* A further expansion of the problem leads to the theorem that *if a locus symmetrical with respect to the origin contains one term of even degree, all the terms are of even degree.*

CALCULUS.

239. Proposed by L. H. MacDONALD, A. M., Ph. D., Sometime Tutor in the University of Cambridge, Jersey City, N. J.

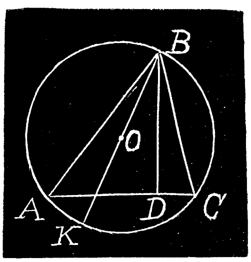
Of all triangles inscribed in a circle, find that which has the greatest perimeter.

Solution by C. N. SCHMALL, 89 Columbia Street, New York City, and REV. J. H. MEYER, S. J., Augusta, Ga.

Let ABC be the required triangle; O, the center of the given circle; BK, $=2r$, the diameter of the circle; $\angle BAC=\phi$, and $\angle BCA=\psi$. Then the perimeter, p, $=AB+BC+AC=$maximum... (1).

But $AB.BC=2r.BD$, whence $BC=2r\dfrac{BD}{AB}=2r\sin\phi$.

Also $\dfrac{AB}{BC}=\dfrac{\sin\psi}{\sin\phi}$, whence $AB=BC\dfrac{\sin\psi}{\sin\phi}$, which, from the previous equation, $=2r\sin\psi$. We also have, from the law of signs,



$$AC=\frac{\sin(\phi+\psi)}{\sin\phi}BC=2r\sin(\phi+\psi).$$

Substituting in (1), we have

$$p=2r[\sin\phi+\sin\psi+\sin(\phi+\psi)]\ldots(2).$$

Hence, for a maximum or minimum,

$$\frac{\partial p}{\partial\phi}=\cos\phi+\cos(\phi+\psi)=0, \text{ and } \frac{\partial p}{\partial\psi}=\cos\psi+\cos(\phi+\psi)=0.$$

Hence, $\cos\phi=\cos\psi$ and $\phi=\psi$. Since $\cos\phi+\cos(\phi+\psi)=0$, we have, by substitution, $\cos\phi+\cos 2\phi=0$, or $\cos\phi+2\cos^2\phi-1=0$, whence, $\cos\phi=\frac{1}{2}$, and therefore $\phi=60°=\psi$.

Hence, the triangle is equilateral.

It is easy to show that $\dfrac{\partial^2 p}{\partial\phi^2}.\dfrac{\partial^2 p}{\partial\psi}>\dfrac{\partial^2 p}{\partial\phi\,\partial\psi}$, and that, therefore, the triangle is a maximum.

Also solved by A. F. Carpenter, G. B. M. Zerr, A. H. Holmes, J. Scheffer, and G. W. Greenwood.

Professors Zerr, Greenwood, and Scheffer, and Mr. Holmes denoted the angles at the center subtended by the sides, by 2θ, 2ϕ, and 2ψ, and showed that these angles are equal to 120° each. Professor Carpenter showed that the inscribed triangle with one side constant and of maximum perimeter is isosceles, and then showed that of all isosceles triangles inscribed in a circle, the equilateral has the maximum perimeter.

240. Proposed by L. MORDELL, Philadelphia, Pa.

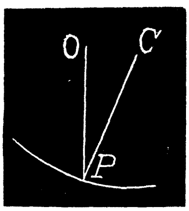
Show that the osculating conic of the catenary $y=c\cosh\frac{x}{c}$ at the point for which $y=\frac{c\sqrt{10}}{2}$ is a parabola.

I. Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let O be the center of curvature of the point considered, and C the center of the conic of closed contact, then (*vide* Joseph Edwards' *Differential Calculus*, where the problem is proposed as an exercise):

$$\frac{\cos\phi}{R}=\frac{1}{\rho}-\frac{\partial\phi}{\partial s}, \text{ and } \phi=\tan^{-1}\tfrac{1}{3}.\frac{\partial\rho}{\partial s},$$

where $\phi=\angle OPC$, $OP=\rho=$radius of curvature, s an arc of the given curve, and $PC=R$. From



$$y=\frac{c}{2}(e^{x/c}+e^{-(x/c)}), \text{ we find } \frac{\partial y}{\partial x}=\tfrac{1}{2}(e^{x/c}-e^{-(x/c)}),$$

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{2c}(e^{x/c}+e^{-(x/c)}, \quad \frac{\partial s}{\partial x}=\tfrac{1}{2}(e^{x/c}+e^{-(x/c)}).$$

$$\therefore \rho=\left[\left(\frac{\partial s}{\partial x}\right)^3\Big/\left(\frac{\partial^2 y}{\partial x^2}\right)\right]=\frac{c}{4}(e^{x/c}+e^{-(x/c)})^2. \quad \therefore \frac{\partial\rho}{\partial s}=\frac{\partial\rho}{\partial x}\cdot\frac{\partial x}{\partial s}=e^{x/c}-e^{-(x/c)}$$

therefore, $\phi=\tan^{-1}\dfrac{e^{x/c}-e^{-(x/c)}}{3}$, or, $e^{x/c}-e^{-(x/c)}=3\tan\phi$.

$$\therefore \frac{\partial\phi}{\partial x}=\frac{(e^{x/c}+e^{-(x/c)})\cos^2\phi}{3c}. \quad \therefore \frac{\partial\phi}{\partial s}=\frac{\partial\phi}{\partial x}\cdot\frac{\partial x}{\partial s}=\frac{2}{3c}\cos^2\phi.$$

For the value $y=\frac{c}{2}\sqrt{10}$, we find $\tan\phi=\frac{1}{3}\sqrt{6}$, $\cos^2\phi=\frac{3}{5}$, $\rho=\frac{5c}{2}$, $\frac{\partial\phi}{\partial s}=\frac{2}{5c}$; substituting in $\frac{\cos\phi}{R}=\frac{1}{\rho}-\frac{\partial\phi}{\partial s}$, we find $\frac{\sqrt{\frac{3}{5}}}{R}=\frac{2}{\rho c}-\frac{2}{\rho c}=0$; $\therefore R=\infty$, and since the conic, the center of which is at an infinite distance, is a parabola, the assertion is proved.